

# Package: kantorovich (via r-universe)

August 21, 2024

**Type** Package

**Title** Kantorovich Distance Between Probability Measures

**Version** 3.2.0

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**Description** Computes the Kantorovich distance between two probability measures on a finite set. The Kantorovich distance is also known as the Monge-Kantorovich distance or the first Wasserstein distance.

**License** GPL-3

**URL** <https://github.com/stla/kantorovich>

**BugReports** <https://github.com/stla/kantorovich/issues>

**Depends** R (>= 4.0.0)

**Imports** CVXR, gmp, lpSolve, methods, rccd, Rglpk, slam, utils, ompr, ompr.roi, ROI.plugin.glpk

**Suggests** knitr, rmarkdown, testthat (>= 3.0.0)

**VignetteBuilder** knitr

**Encoding** UTF-8

**RoxygenNote** 7.3.1

**SystemRequirements** GMP (<https://gmplib.org/>)

**Repository** <https://stla.r-universe.dev>

**RemoteUrl** <https://github.com/stla/kantorovich>

**RemoteRef** HEAD

**RemoteSha** ef17d21566791c656d1f455344b736855e57e83b

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kantorovich-package     *Kantorovich Distance Between Probability Measures*

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## Description

Computes the Kantorovich distance between two probability measures on a finite set, also known as the earth mover's distance. The Kantorovich distance is not a "unique" distance: it is defined by a given distance on the two finite sets (generally equal). Note that the default distance is the 0-1 distance and with this choice the Kantorovich computation is totally useless (see the vignette). Computing the Kantorovich distance is a linear programming problem, and several methods are provided in the package. In particular there is an exact method available when the probability weights are rational numbers and when the distances are rational numbers as well. A benchmark suggests that the faster methods are those using the 'CVXR' package.

To learn more, start with the vignettes: `browseVignettes(package="kantorovich")`.

If you encounter a bug, or if you have a suggestion to improve the package, please file an issue on the Github repo <https://github.com/stla/kantorovich>.

## Details

Package:   kantorovich  
 Type:     Package  
 Version:   3.1.0  
 Date:     2023-08-22  
 License:   GPL-3

## Author(s)

Stéphane Laurent

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edistances	<i>Extremal distances</i>
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**Description**

Compute the distances at the extreme joinings.

**Usage**

```
edistances(mu, nu, dist = NULL, ...)
```

**Arguments**

mu	(row margins) probability measure in numeric or bigq/character mode
nu	(column margins) probability measure in numeric or bigq/character mode
dist	function or matrix, the distance to be minimized on average. If NULL, the 0-1 distance is used.
...	arguments passed to dist

**Value**

A list with two components: the extreme joinings in a list and the distances in a vector.

**Note**

This function, called by [kantovich](#), is rather for internal purpose.

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ejoinings	<i>Extreme joinings</i>
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**Description**

Return extreme joinings between mu and nu.

**Usage**

```
ejoinings(mu, nu, zeros = FALSE)
```

**Arguments**

mu	(row margins) probability measure in numeric or bigq/character mode
nu	(column margins) probability measure in numeric or bigq/character mode
zeros	logical; in case when mu and nu have different lengths, set FALSE to remove lines or columns full of zeros

**Value**

A list containing the extreme joinings (matrices).

**Examples**

```
mu <- nu <- c(0.5, 0.5)
ejoinings(mu, nu)
# use exact arithmetic
library(gmp)
mu <- nu <- as.bigq(c(0.5,0.5))
ejoinings(mu, nu)
# different lengths example
mu <- setNames(as.bigq(c(1,2,4), 7), c("a", "b", "c"))
nu <- setNames(as.bigq(c(3,1), 4), c("b", "c"))
ejoinings(mu, nu)
```

---

kantorovich

*Kantorovich distance*


---

**Description**

Compute the Kantorovich distance between two probability measures on a finite set.

**Usage**

```
kantorovich(mu, nu, dist = NULL, details = FALSE, ...)
```

**Arguments**

mu	(row margins) probability measure in numeric or bigq/character mode
nu	(column margins) probability measure in numeric or bigq/character mode
dist	function or matrix, the distance to be minimized on average; if NULL, the 0-1 distance is used.
details	prints the joinings achieving the Kantorovich distance and returns them in the "joinings" attribute of the output
...	arguments passed to dist (only if it is a function)

**Details**

The function firstly computes all the extreme joinings of mu and nu, then evaluates the average distance for each of them, and then returns the minimal one.

**Value**

The Kantorovich distance between mu and nu.

**Examples**

```

mu <- c(1/7, 2/7, 4/7)
nu <- c(1/4, 1/4, 1/2)
kantorovich(mu, nu)
library(gmp)
mu <- as.bigq(c(1,2,4), 7)
nu <- as.bigq(c(1,1,1), c(4,4,2))
kantorovich(mu, nu)
mu <- c("1/7", "2/7", "4/7")
nu <- c("1/4", "1/4", "1/2")
kantorovich(mu, nu, details=TRUE)

```

---

kantorovich\_CVX

*Computes Kantorovich distance with CVX*


---

**Description**

Kantorovich distance using the CVXR package

**Usage**

```

kantorovich_CVX(
  mu,
  nu,
  dist,
  solution = FALSE,
  stop_if_fail = TRUE,
  solver = "ECOS",
  ...
)

```

**Arguments**

mu	(row margins) probability measure in numeric mode
nu	(column margins) probability measure in numeric mode
dist	matrix defining the distance to be minimized on average
solution	logical; if TRUE the solution is returned in the "solution" attributes of the output
stop_if_fail	logical; if TRUE, an error is returned in the case when no solution is found; if FALSE, the output of <a href="#">psolve</a> is returned with a warning
solver	the CVX solver, passed to <a href="#">psolve</a>
...	other arguments passed to <a href="#">psolve</a>

**Examples**

```
x <- c(1.5, 2, -3)
mu <- c(1/7, 2/7, 4/7)
y <- c(4, 3.5, 0, -2)
nu <- c(1/4, 1/4, 1/4, 1/4)
M <- outer(x, y, FUN = function(x, y) abs(x - y))
kantorovich_CVX(mu, nu, dist = M)
```

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kantorovich_glpk	<i>Computes Kantorovich distance with GLPK</i>
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**Description**

Kantorovich distance using the Rglpk package

**Usage**

```
kantorovich_glpk(mu, nu, dist, solution = FALSE, stop_if_fail = TRUE, ...)
```

**Arguments**

mu	(row margins) probability measure in numeric mode
nu	(column margins) probability measure in numeric mode
dist	matrix defining the distance to be minimized on average
solution	logical; if TRUE the solution is returned in the "solution" attributes of the output
stop_if_fail	logical; if TRUE, an error is returned in the case when no solution is found; if FALSE, the output of <a href="#">Rglpk_solve_LP</a> is returned with a warning
...	arguments passed to <a href="#">Rglpk_solve_LP</a>

**Examples**

```
x <- c(1.5, 2, -3)
mu <- c(1/7, 2/7, 4/7)
y <- c(4, 3.5, 0, -2)
nu <- c(1/4, 1/4, 1/4, 1/4)
M <- outer(x, y, FUN = function(x, y) abs(x - y))
kantorovich_glpk(mu, nu, dist = M)
```

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kantorovich\_lp      *Computes Kantorovich distance with lp\_solve*

---

**Description**

Kantorovich distance using the lpSolve package

**Usage**

```
kantorovich_lp(mu, nu, dist, solution = FALSE, lp.object = FALSE, ...)
```

**Arguments**

mu	(row margins) probability measure in numeric mode
nu	(column margins) probability measure in numeric mode
dist	matrix defining the distance to be minimized on average
solution	logical, to use only if lp.object=FALSE; if TRUE the solution is returned in the "solution" attributes of the output
lp.object	logical, if FALSE, the output is the Kantorovich distance; if TRUE, the output is a <a href="#">lp.object</a>
...	arguments passed to <a href="#">lp</a>

**Examples**

```
x <- c(1.5, 2, -3)
mu <- c(1/7, 2/7, 4/7)
y <- c(4, 3.5, 0, -2)
nu <- c(1/4, 1/4, 1/4, 1/4)
M <- outer(x, y, FUN = function(x, y) abs(x - y))
kantorovich_lp(mu, nu, dist = M)
```

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kantorovich\_ompr      *Computes Kantorovich distance with 'ompr'*

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**Description**

Kantorovich distance using the ompr package

**Usage**

```
kantorovich_ompr(mu, nu, dist, solution = FALSE, stop_if_fail = TRUE)
```

**Arguments**

mu	(row margins) probability measure in numeric mode
nu	(column margins) probability measure in numeric mode
dist	matrix defining the distance to be minimized on average
solution	logical; if TRUE the solution is returned in the "solution" attributes of the output
stop_if_fail	logical; if TRUE, an error is returned in the case when no solution is found; if FALSE, the output of <code>solve_model</code> is returned with a warning

**Note**

The glpk solver is the one used to solve the problem.

**Examples**

```
x <- c(1.5, 2, -3)
mu <- c(1/7, 2/7, 4/7)
y <- c(-4, 3.5, 0)
nu <- c(1/4, 1/4, 1/2)
M <- outer(x, y, FUN = function(x, y) abs(x - y))
kantorovich_ompr(mu, nu, dist = M)
```

---

names.bigq

*Names for bigq vectors*


---

**Description**

Names for bigq vectors

**Usage**

```
## S3 method for class 'bigq'
names(x)
```

**Arguments**

x a bigq vector

**Value**

the names of x



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